Axial Compressive Strength of Carbon Fiber with Tensile Strength Distribution

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SYNOPSIS

Measuring the fiber lengths of the broken pieces and estimating the mean tensile strength from the length just before the final fragment length in tension, efforts were made to estimate the axial compressive strengths of carbon fibers when the tensile strength varies with the length. The estimated compressive strength of carbon fibers decreases with increasing temperature. This decrease in compressive strength may be accounted for by a decrease in the radial compressive force owing to a decrease in the residual thermal stress and a decrease in Young's modulus of the resin matrix. There is a linear relationship between the estimated compressive strength and radial compressing force in the temperature range from room temperature to 80°C. The real compressive strength of the fibers, determined by extrapolating this straight line until the radial compressing force is zero, is about 20% higher than the compressive strength estimated by assuming that the tensile strength is uniform. It is approximately 10–50% of tensile strength. A linear relationship between the fiber axial compressive strength and compressive strength of the unidirectional composites is found. The experimental values agree with the values calculated by the rule of mixtures.

INTRODUCTION

The relation between the mechanical properties of composites in tension and those of the fibers used as reinforcements have been studied both experimentally and theoretically in fairly great detail. However, in spite of the fact that the compressive characteristics of composites depend on the characteristics of reinforcing fibers, there is almost no study of the mechanical properties of composites in compression. A few efforts to determine the compressive characteristics of reinforcing fibers have been made,[1–4] but there is still no decisive method.

In the preceding paper,[5] we reported that if a sufficiently long fiber is embedded in the neighborhood of the surface of rectangular beam and the system is subjected to a tensile (or compressive) strain rather than a fiber ultimate strain according to the bending method, the fiber eventually breaks into many pieces. Measuring the length of the broken pieces, the axial compressive strength of carbon fibers can be estimated when uniform strength of the fiber is assumed. It was found that the estimated compressive strength of carbon fiber varies with the kinds of fibers and is approximately 10–60% of tensile strength.

It is generally known that the tensile strength of carbon and glass fibers varies with fiber length. In this article, efforts were made to accurately determine the fiber axial compressive strength taking the variation of fiber tensile strength into consideration. Namely, first, the distribution of fiber tensile strength is measured. Second, the fiber fragment length in tension and compression is measured according to the experimental procedure provided in our preceding paper.[5] Furthermore, if the length just before the final fragment length in tension is estimated, the mean tensile strength of fiber with an estimated length is determined. Finally, by using the lengths of the broken pieces in tension and compression and the estimated mean tensile

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strength, the fiber axial compressive strength can be estimated.

EXPERIMENTAL

Procedure

In the preceding paper,[5] when uniform tensile strength of the fiber is assumed, the axial compressive strength $(\sigma_f)_c$ of fiber was given by:

$$(\sigma_f)_c = (\sigma_f)_t \cdot \frac{(l_c)_c}{(l_c)_t}, \tag{1}$$

where $(\sigma_f)_t$ is the fiber tensile strength, $(l_c)_c$ is the critical fiber length in compression, and $(l_c)_t$ is the critical fiber length in tension.

As stated above, the tensile strength of carbon and glass fibers varies with fiber length. The tensile strength of brittle fibers such as a carbon fiber is generally affected by partial flaws. In general, the tensile strength of such fibers is represented by a chain model. This model represents a fiber by a chain consisting of n pieces of equal links. Applying the Weibull distribution function, probability $g(\sigma)$ in which a chain of n links will break at stress σ can be expressed as[6]:

$$g(\sigma) = n \cdot m \cdot \sigma_o^{-1}\left(\frac{\sigma - \sigma_p}{\sigma_o}\right) \times \exp\left[-n\left(\frac{\sigma - \sigma_p}{\sigma_o}\right)^m\right], \tag{2}$$

where m, σ_o, and σ_p are the Weibull parameters. Also, the mean tensile strength $(\bar{\sigma}_{f,L})_t$ of the fiber at length L is given as[6,7]:

$$(\bar{\sigma}_{f,L})_t = \sigma_p + \left\{\frac{\sigma_o}{(L/L_l)^{1/m}}\right\} \cdot \Gamma\left(\frac{m+1}{m}\right), \tag{3}$$

where Γ is the complete gamma function. L_l is the length of link (gauge length L/number of the links) consisting of the fibers.

As stated above, if a sufficiently long fiber is embedded in the resin matrix and the system is elongated, the fiber eventually breaks into many pieces.[5-8] We previously proposed a method to accurately determine the critical fiber length taking the tensile strength distribution of fiber into consideration.[5,7] Namely, considering all the final fiber fragment lengths obtained for the system, average value $(\bar{l}_c)_t$ of critical fiber length in tension for the system is given by[7,9-11]:

$$(\bar{l}_c)_t = \tfrac{4}{3}(\bar{l})_t, \tag{4}$$

where $(\bar{l})_t$ is the mean fragment length.

On the other hand, when the above-mentioned specimen is compressed in the direction of the fiber axis, we assumed that the yield shear strengths are the same value as that in tension, but with an opposite working direction. Therefore, if the specimen is compressed, the fiber eventually breaks into many pieces in the same manner as in the case of tension. When the tensile strength of the fiber varies with fiber length, on the average, for the whole system, the relationship between mean critical fiber length $(\bar{l}_c)_c$ in compression and mean fragment length $(\bar{l})_c$ in compression results in the following equation:

$$(\bar{l}_c)_c = \tfrac{4}{3}(\bar{l})_c. \tag{5}$$

According to the procedure mentioned in the preceding paper,[5] in the case in which the fiber tensile strength is variable, the fiber axial compressive strength is given by:

$$(\sigma_f)_c = (\bar{\sigma}_{f,L})_t \cdot \frac{(\bar{l}_c)_c}{(\bar{l}_c)_t} = (\sigma_{f,L})_t \cdot \frac{(\bar{l})_c}{(\bar{l})_t}. \tag{6}$$

When all the fragments are reduced to less than the critical fiber length, any further elongation of specimen will not break the fiber. When the fiber finally breaks into many pieces, we previously proposed that this considerable tensile strength is the tensile strength at the length just before the final fragment lengths.[7]

Let $(\bar{l})$ represents the average value of final fragment length, $K(\bar{l})$ the first embedded length, and $k(\bar{l})$ the length just before the length $(\bar{l})$. Thus, average value $(\bar{L})$ of length just before the fragment length with $\bar{l}$ is given by:

$$\bar{L} = \left(\frac{4}{3}\right)\bar{l} + \sum_{k=3}^{K-1} k(\bar{l}) \cdot \frac{4}{k-1}\left(\frac{1}{3}\right)^{k-1} + K(\bar{l}) \times \frac{2}{K-1}\left(\frac{1}{3}\right)^{K-2}, \quad k = 2, 3, \ldots, K. \tag{7}$$

Accordingly, the mean tensile strength of a fiber can be determined by subsituting the average value $\bar{L}$ of length into eq. (3).

Therefore, first, the fiber tensile strength is measured and Weibull parameters m, σ_o, and σ_p and the length of link L_l of the fiber used in the system are determined. Second, the mean fragment length $(\bar{l})_t$ in tension is obtained and substituted into eq. (6). Moreover, using eq. (7) with value $(\bar{l})_t$, average value $\bar{L}$ of the length just before the fragment length $(\bar{l})_t$ can be calculated. Substituting value $\bar{L}$ into eq. (3), the mean tensile strength $(\bar{\sigma}_{f,L})_t$ of fiber can be determined. We substitute the value $(\bar{\sigma}_{f,L})_t$ into eq. (6). On the other hand, if the specimen is compressed, mean fragment length $(\bar{l})_c$ in compression is obtained and substituted into eq. (6). In case the fiber tensile strength is variable, the fiber axial compressive strength can finally be estimated.

Preparation of Specimen

The carbon fibers and resin used in preparation of the specimens are the same as those described in the preceding paper.[5] Namely, the fibers used were pitch-based carbon fibers (Tonen, PCT and PGM, experimental samples of Tonen 10.1 μm in diameter) and PAN-based carbon fibers (Toray, Torayca T-300 and M-40, 7.1 μm in diameter). Moreover, carbonized (higher strength type, PCT and T-300) and graphitized fibers (higher modulus type, PGM and M-40) were respectively used.

First, tensile tests were performed using 5-, 10-, 15-, 20-, 25-, and 100-mm gauge lengths to obtain distribution curves of tensile strength for the carbon fibers. The instrument used was Tensilon UTM-1 type (Orientec), strain rate 0.05 mm/mm/min, and the number of fibers tested was 100 pieces for each gauge length. The results obtained by the fiber tensile strength distribution tests were used to determine the number of links of eq. (2) and Weibull parameters m, σ_o, and σ_p.

Second, rectangular specimens, in which a long fiber was embedded at a constant tension in the neighborhood of the surface, were prepared for measuring fragment length under the same conditions as reported in the preceding paper.[5] Epoxy resin (Epikote 828, Yuka Shell), 100 parts, was mixed with 10 parts of an amine-hardening agent (S-Cure 661, Kayaku Nuri). The mixture agitated thoroughly and then defoamed. This mixture was poured into a mold holding a fiber at a constant tension in the neighborhood of the surface of rectangular specimen and subjected to curing at 65°C for 17 h and postcuring at 140°C for 5 h.

The specimens prepared in this manner were submitted to measurement of fragment length; each specimen was subjected to a tensile (or compressive) strain of 4% at a drop rate of the upper heads of 10 mm/min using the four-point bending method under the same conditions as reported in the preceding paper.[5]

As indicated before,[5] the fiber buckles at a temperature range higher than 100°C. Accordingly, the conditions for application of the above-mentioned eq. (1) are not satisfied. In this experiment, to invesitgate the effect of radially compressing force on the fiber axial compressive strength, measurements were made at intervals of 20°C from 20–100°C. Over 1,000 fragments were examined for each experimental condition.

RESULTS AND DISCUSSION

The typical strength distributions of carbon fibers used in the experiments are shown in Figure 1. The solid lines in the figure represent theoretical values obtained by substituting Weibull parameters m, σ_o, and σ_p and the length of links L_l given in Table I into eq. (2). The number of the links is calculated by gauge length/length of links.

The relationship between the logarithm of gauge length and the logarithm of the mean tensile strength is shown in Figure 2. The solid lines in the figure represent theoretical values calculated by substituting Weibull parameters and the length of links into eq. (3). The theoretical values obtained by eq. (3) employing the values of Weibull parameters and the length of links given in Table I agree with the measured values. Therefore, the values of Weibull parameters and the length of links thus determined are appropriate. Moreover, the constant value of mean tensile strength within the range of shorter gauge length shows that the tensile strength of fiber is uniform in length range shorter than the link length.

The relationships between both the mean fragment length $(\bar{l})_t$ in tension and $(\bar{l})_c$ in compression of composite systems including carbon fibers having tensile strength distributions and the temperature are shown in Figures 3 and 4, respectively. Average value $\bar{L}$ of the length just before the length $(\bar{l})_t$ from the mean fragment length $(\bar{l})_t$ in tension according to eq. (7) is calculated. Furthermore, the mean tensile strength $(\bar{\sigma}_{f,L})_t$ of carbon fibers is determined by substituting value $\bar{L}$ into eq. (3).

Figure 5 shows the relationship between the value of fiber axial compressive strength estimated from the mean fragment length $(\bar{l})_t$ (Fig. 3) in tension, the mean fragment length $(\bar{l})_c$ (Fig. 4) in compres-

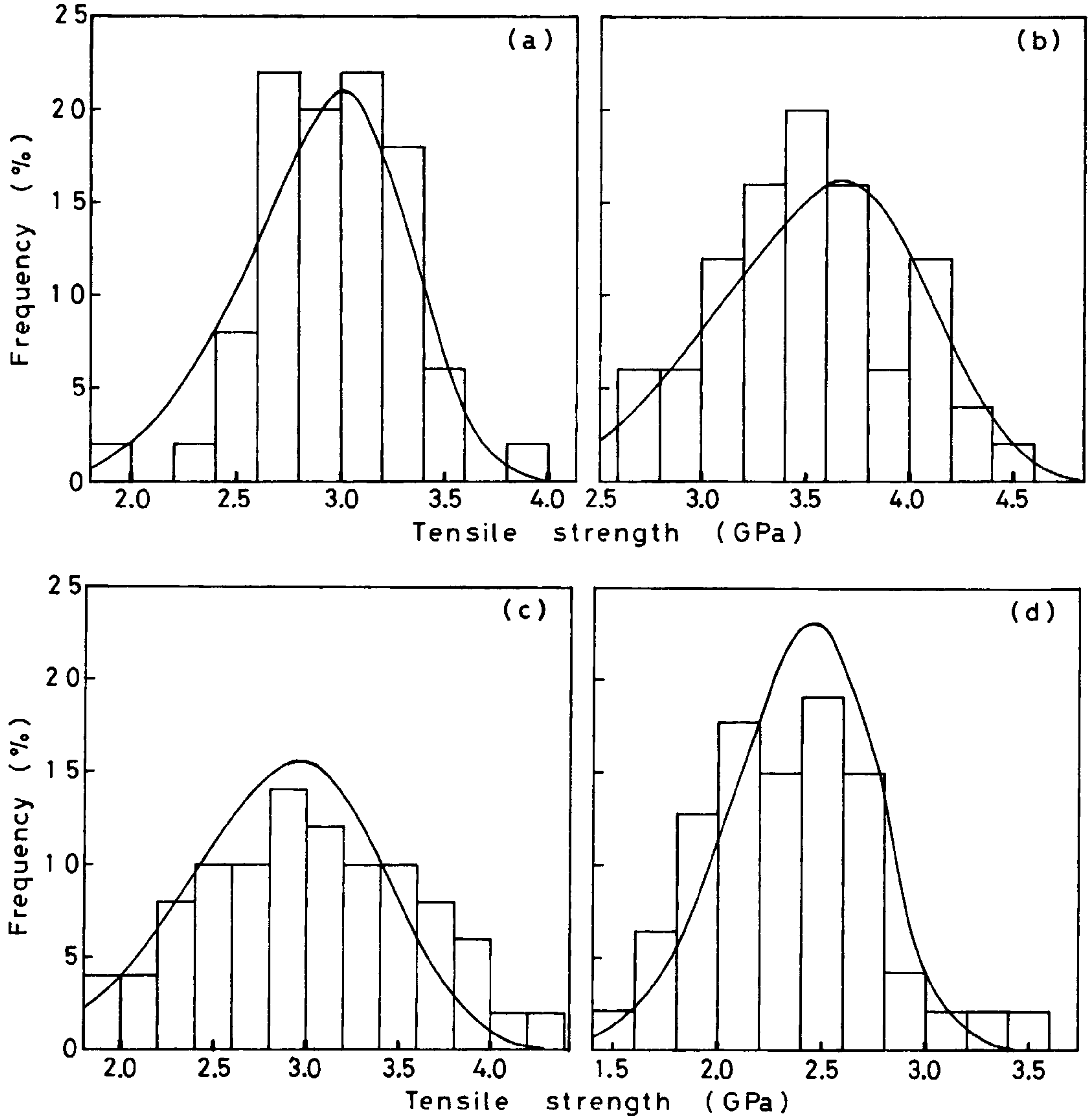

Figure 1 Strength distribution of carbon fibers (test length: 25 mm). (a), pitch-based carbonized fiber, PCT; (b), pitch-based graphitized fiber, PGM; (c), PAN-based carbonized fiber, T-300; (d), PAN-based graphitized fiber, M-40.

sion, and the mean tensile strength $(\bar{\sigma}_{f,L})_t$ of the fiber according to eq. (6) and the temperature. With these systems, in the same manner as in the case in which the tensile strength of the fiber is uniform,[5] the estimated compressive strength decreases with increasing temperature. As indicated before,[5] it is conceivable that the radial force compressing the fiber decreased by both the decrease in residual thermal stress and Young's modulus of the resin matrix with increasing temperature may cause a temperature dependence of the estimated value of the fiber axial compressive strength.

Glass transition temperature T_g of epoxy resin matrix used in this experiment is about 80°C. Therefore, it is conceivable that the conditions for application of the above-mentioned eq. (6), while deriving by the use of Kelly and Tyson's theory,[12,13] is not satisfied above 80°C. Accordingly, further details are discussed for the results obtained at a temperature range lower than 80°C.

When a fiber is embedded in the resin and the system is allowed to cure, the thermal stress $(P)_T$ working perpendicularly on the fiber–resin interface is approximately given by the following equation[8,14]:

Table I Statistical Values of Tensile Strength for Carbon Fibers

Carbon Fiber	m	σ_o (GPa)	σ_p (GPa)	Length of Links L_l (mm)
Pitch-Based				
Carbonized fiber PCT	7.32	4.38	0.429	0.625
Graphitized fiber PGM	7.13	5.45	0.426	0.714
PAN-Based				
Carbonized fiber T-300	5.28	5.02	0.502	0.714
Graphitized fiber M-40	5.35	3.99	0.800	0.286

PCT and PGM are Tonen's pitch-based high-performance carbon fibers. They are the materials under development.

$$(P)_T \approx \frac{\alpha_m \cdot E_m \cdot \Delta T}{1 + \nu_m}, \tag{8}$$

where α is the thermal expansion coefficient, E is the Young's modulus, ν is the Poisson's ratio, ΔT is the difference in temperature from molding temperature, and the subscript m denotes matrix. The thermal stress $(P)_T$ was then determined by using eq. (8).[5] Figure 6 shows the relationship between the estimated compressive strength (Fig. 5) and the thermal stress obtained through the function of the temperature. The estimated compressive strength of carbonized and graphitized fibers decreases linearly with decreasing thermal stress.

As indicated in the preceding paper,[5] it is conceivable that the real compressive strength of carbonized and graphitized fibers is the strength when the radial compressing force, i.e., the residual thermal stress in this experiment, is zero. Accordingly, we considered the value obtained by extrapolating the straight line in Figure 6 to $(P)_T = 0$ as the real compressive strength of carbonized and graphitized fibers and showed those values in Table II. For comparison, the compressive strength estimated from the tensile strength obtained by employing the gauge

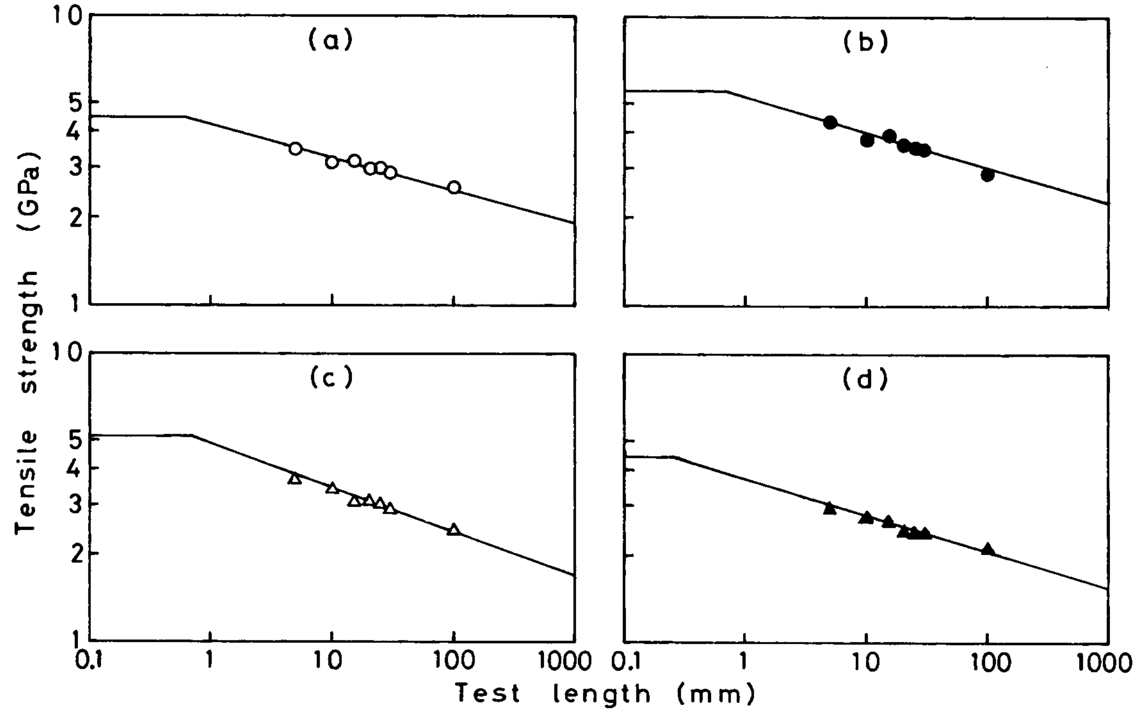

Figure 2 Relation between test length and tensile strength. (a), pitch-based carbonized fiber, PCT; (b), pitch-based graphitized fiber, PGM; (c), PAN-based carbonized fiber, T-300; (d), PAN-based graphitized fiber, M-40.

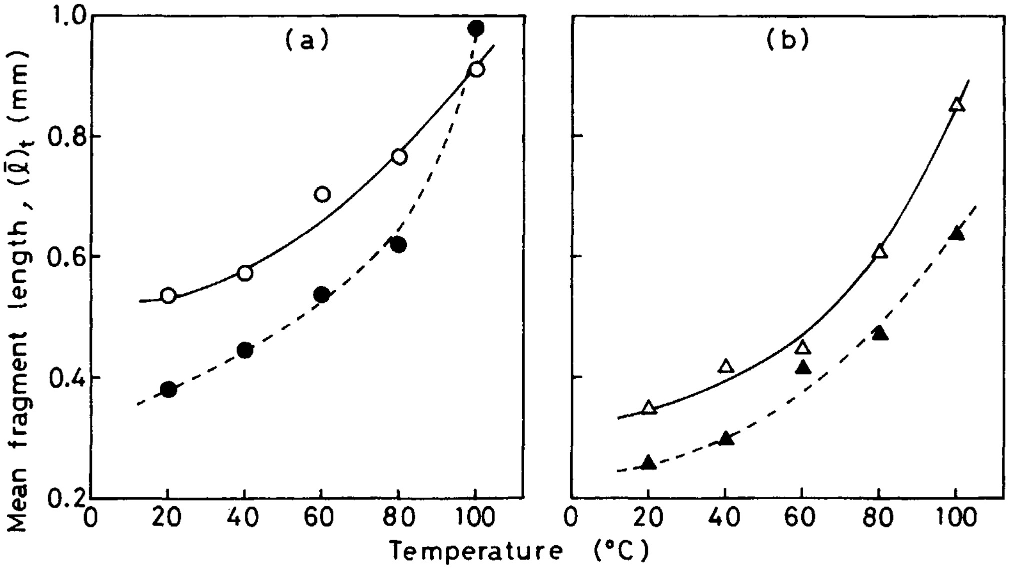

Figure 3 Relation between temperature and mean fragment length in tension. (a), pitch-based carbon fiber; (b), PAN-based carbon fiber; (○), carbonized fiber, PCT; (●), graphitized fiber, PGM; (△), carbonized fiber, T-300; (▲), graphitized fiber, M-40.

length of 5 mm, while assuming that its strength is uniform, is also shown in Table II. Furthermore, the mean tensile strength calculated from the mean fragment length $(\bar{l})_t$ at 20°C according to eqs. (3) and (7) and the ratio of compressive strength to the tensile strength are also shown in Table II. The estimated real compressive strength of PAN-based carbonized fiber (higher strength type, T-300) is the highest and pitch-based carbonized fiber (PCT) follows. For both PAN- and pitch-based fibers, the estimated compressive strength of graphitized fiber (higher modulus type, M-40 and PGM) is always less than half of that of carbonized fiber (higher strength type, T-300 and PCT). In comparison with the compressive strength estimated by assuming that the tensile strength of the fiber is uniform, the compressive strength estimated in this experiment, taking the variation of tensile strength into consideration, is approximately 15–22% higher. Furthermore, the ratio of compressive strength to the tensile

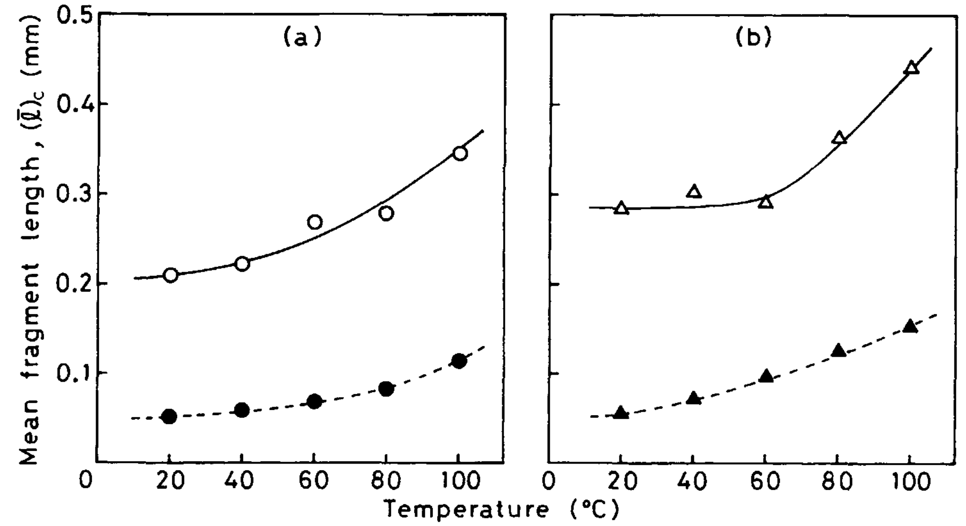

Figure 4 Relation between temperature and mean fragment length in compression. (a), pitch-based carbon fiber; (b), PAN-based carbon fiber; (○), carbonized fiber, PCT; (●), graphitized fiber, PGM; (△), carbonized fiber, T-300; (▲), graphitized fiber, M-40.

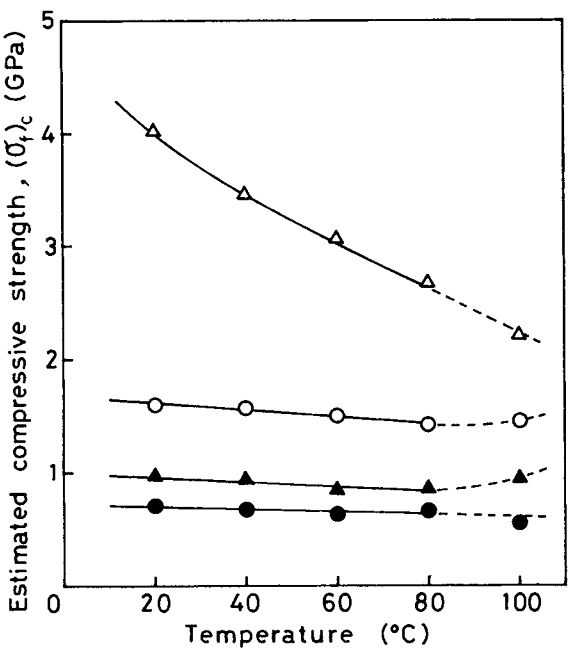

Figure 5 Relation between temperature and estimated compressive strength. (○), pitch-based carbonized fiber, PCT; (●), pitch-based graphitized fiber, PGM; (△), PAN-based carbonized fiber, T-300; (▲), PAN-based graphitized fiber, M-40.

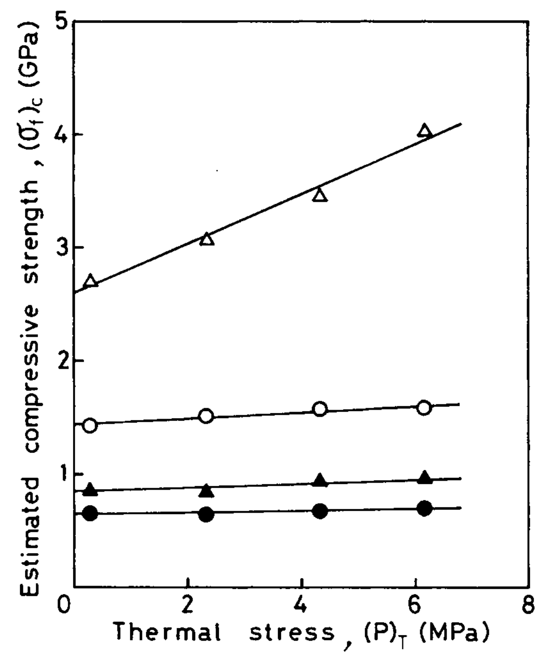

Figure 6 Relation between thermal stress and estimated compressive strength. (○), pitch-based carbonized fiber, PCT; (●), pitch-based graphitized fiber, PGM; (△), PAN-based carbonized fiber, T-300; (▲), PAN-based graphitized fiber, M-40.

strength is approximately 10–50%, but its ratio depends on the kinds of fibers.

We will discuss now some applications of the method for measuring fiber axial compressive strength. The relationship between the fiber axial compressive strength and compressive strength of unidirectional composites is shown in Figure 7. Test specimens of composites were cut and tested in accordance with ASTM D 3410-75. Fiber volume fraction V_f was set at 0.6. The compressive strength of composites is obtained at 20°C. The compressive strength of composites increases almost linearly as the fiber axial compressive strength increases. Then the experimental values were compared with the values calculated by the following equation, while assuming that a rule of mixtures also holds for compressive strength.

$$(\sigma_c)_c = (\sigma_f)_c \cdot V_f + \sigma_m \cdot V_m \approx (\sigma_f)_c \cdot V_f, \quad (9)$$

where $(\sigma_c)_c$ is the compressive strength of composites, $(\sigma_f)_c$ is the fiber compressive strength, σ_m is the compressive strength of the matrix, V_f is the fiber volume fraction, and V_m is the matrix volume fraction.

Table II Estimated Compressive and Tensile Strength of Carbon Fiber

	Pitch-Based		PAN-Based	
Carbon Fiber	Carbonized Fiber PCT	Graphitized Fiber PGM	Carbonized Fiber T-300	Graphitized Fiber M-40
Estimated compressive strength $(\sigma_f)_c$ (GPa)[a]	1.25	0.54	2.06	0.78
Estimated compressive strength $(\sigma_f)_c$ (GPa)[b]	1.42	0.65	2.62	0.86
Tensile strength $(\sigma_f)_t$ (GPa)[c]	4.15	5.38	5.04	4.01
$(\sigma_f)_c/(\sigma_f)_t$ (%)	34.2	12.1	52.0	21.4

[a] Values estimated from tensile strength, while assuming that its strength is uniform.
[b] Values estimated in this study.
[c] Mean tensile strength calculated from mean fragment length.

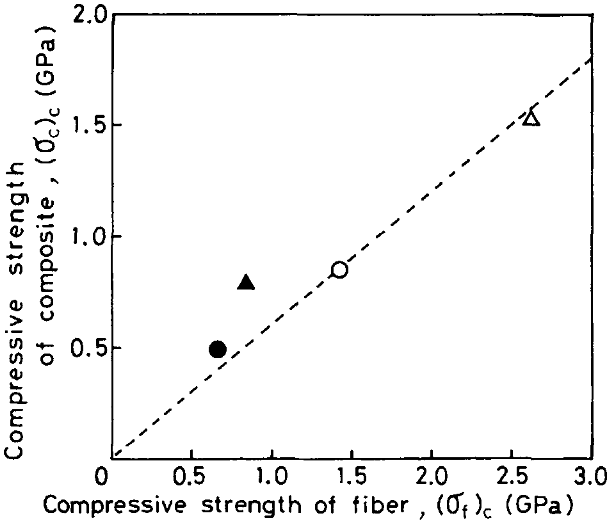

Figure 7 Relation between compressive strength of carbon fiber and compressive strength of unidirectional composite. (○), pitch-based carbonized fiber, PCT; (●), pitch-based graphitized fiber, PGM; (△), PAN-based carbonized fiber, T-300; (▲), PAN-based graphitized fiber, M-40.

The broken line in Figure 7 shows the values calculated by eq. (9). The experimental values agree with the calculated ones. Therefore, it is found that the rule of mixtures holds for the compressive strength of unidirectional composites in the same manner as the tensile strength. The fact that the rule of mixture is established between the axial compressive strength of carbon fiber estimated according to the method proposed in this study and the compressive strength of unidirectional composites reinforced with the carbon fibers supports the reliability of the method for measuring compressive strength.

CONCLUSIONS

If a sufficiently long fiber is embedded in the neighborhood of the surface of the rectangular beam and the system is subjected to a tensile (or compressive) strain greater than the fiber ultimate strain according to the bending method, the fiber eventually breaks into many pieces. Measuring the lengths of the broken pieces and estimating the mean tensile strength from the length just before the final fragment length in tension, attempts were made to estimate the fiber axial compressive strength of carbon fibers when the tensile strength varies with the length.

The estimated compressive strength of carbon fibers decreases with increasing temperature. This decrease in compressive strength may be accounted for by a decrease in the radial compressing force owing to a decrease in the residual thermal stress and a decrease in Young's modulus of the resin matrix.

There is a linear relationship between the estimated compressive strength and radial compressing force in a temperature range from room temperature to 80°C. The real compressive strength of the fibers, determined by extrapolating this straight line until the radial compresing force is zero, is about 20% higher than the compressive strength estimated by assuming that the tensile strength is uniform. It is approximately 10–50% of tensile strength.

A linear relationship between the fiber axial compressive strength and compressive strength of the unidirectional composites is found. The experimental values agree with the values calculated by the rule of mixtures. This suggests the reliability of our method for measuring compressive strength.

REFERENCES

1. J. H. Sinclair and C. C. Chamis, Compression Testing of Homogeneous Materials and Composites, ASTM STP 808, ASTM, Philadelphia, PA, 1983, p. 155.
2. W. R. Jones and J. W. Johnson, *Carbon,* **9,** 645 (1971).
3. H. M. Hawthorne and E. Teghtsoonian, *J. Mater. Sci.,* **10,** 41 (1975).
4. S. R. Allen, *J. Mater. Sci.,* **22,** 853 (1987).
5. T. Ohsawa, M. Miwa, M. Kawade, and E. Tsushima, *J. Appl. Polym. Sci.,* **39,** 1733 (1990).
6. M. Miwa, T. Ohsawa, and K. Tahara, *J. Appl. Polym. Sci.,* **25,** 795 (1980).
7. M. Miwa, T. Ohsawa, and A. Tomita, *Kobunshi-ronbunshu,* **41,** 353 (1984).
8. T. Ohsawa, A. Nakayama, M. Miwa, and A. Hasegawa, *J. Appl. Polym. Sci.,* **22,** 3203 (1978).
9. M. J. Folks and W. K. Wong, *Polymer,* **28,** 1309 (1987).
10. A. T. DiBenedetto and L. Nicolais, *Proc. 40th Annual Conf. Reinf. Plast./Compos. Inst., SPI,* Session 21-A, pp. 1–6, 1985.
11. A. T. DiBenedetto and P. J. Lex, *Polym. Eng. Sci.,* **29,** 543 (1989).
12. A. Kelly and W. R. Tyson, *J. Mech. Phys. Solids,* **13,** 329 (1965).
13. A. Kelly and W. R. Tyson, *J. Mech. Phys. Solids,* **14,** 177 (1966).
14. G. Gerard and A. C. Gilbert, *J. Appl. Mech. (ASME),* **24,** 355 (1957).

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